

Shot noise in HTc superconductor quantum point contact system

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Abstract

We study the electrical transport properties of a quantum point contact between a lead and a High Tc superconductor. For this, we use the Hamiltonian approach and non-equilibrium Green functions of the system. The electrical current and the shot noise are calculated with this formalism. We consider $d_{x^2-y^2}$, d_{xy} , $d_{x^2-y^2} + is$ and $d_{xy} + is$ symmetries for the pair potential. Also we explore the s_{+-} and s_{++} symmetries describing the behavior of the ferropnictides superconductors. We found that for d_{xy} symmetry there is not a zero bias conductance peak and for $d + is$ symmetries there is a displacement of the transport properties. From shot noise and current, the Fano factor is calculated and we found that it takes values of effective charge between e and $2e$, this is explained by the diffraction of quasiparticles in the contact. For the s_{+-} and s_{++} symmetries the results show that the electrical current and the shot noise depend on the mixing coefficient, furthermore the effective electric charge can take values between 0 and $2e$, in contrast with the results obtained for s wave superconductors.

Key words: Superconductivity, Andreev reflection, Tunneling phenomena, Nanocontacts

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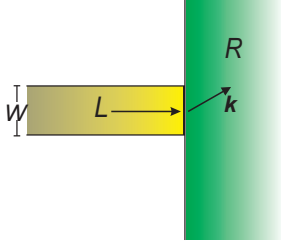


Fig. 1. Scheme of a normal metal lead with width w (L Region) connected to a high Tc superconductor (R Region), θ is the angle of the quasiparticles in the superconductor.

1. Introduction

There are two current noise sources, one is thermal fluctuations, which causes changes in the occupation number and is known as Johnson-Nyquist noise [1]. The other is the discrete behavior of the electric

charge and is known as shot noise [2]. From measures of shot noise (S) and electrical current (I) can be obtained information that usually is not found in conductance measures as the effective electric charge, by means of the ratio between S and I called Fano factor (F) [3]. The shot noise in superconducting systems has attracted attention due to the possibility to obtain effective electric charge equal to $2e$, this has been analyzed for plain junctions and non-isotropic superconductors [4–6]. Nevertheless, the shot noise has not been studied in a high Tc superconductor connected to a quantum point contact, where the differential conductance and the other properties are affected by the size of the electrode. Some studies had shown an isotropic component in the pair potential of HTcS [7–9], this isotropic compound affect the magnitude and phase of the pair potential and the transport properties. Additionally, the recent discovery of high-Tc superconductivity in ferropnictides [10] has stimulated experimental and theoretical studies of these new superconductors. The main feature of these systems is their multiple band structure near of the Fermi level. In a simplified

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scheme, the structure is reduced to two band model, where the symmetry of the pair potential in each band could be different, and experimental evidence has been favorable to the s_{++} or s_{+-} symmetries [10–12].

In the present work we show the results of the electrical current, the shot noise and the Fano factor for a superconducting point contact system using the hamiltonian approach. We analyze these results considering s , d , $d + is$ and s_{+-} symmetries for the pair potential.

2. Theory

One method used to determine the transport properties in superconducting junctions, is the Hamiltonian approach [13]. The main idea of this approach is to write a Hamiltonian that describes every region involved in the junction and a coupling term between those regions, which gives information about the probability of one particle crossing from one region to other. The Hamiltonian approach allows us to determine the transport properties using the non-equilibrium Green functions of the system [14]. We consider a semi-infinite superconductor (R -region) connected to one normal metal lead (L region). Our aim is to find the electrical current, the shot noise and the Fano factor when a voltage V is applied between the lead and the superconducting region. For d wave superconductors, we consider two cases: the first $d_{x^2-y^2}$ symmetry where $\Delta_{x^2-y^2}(\theta) = \Delta_0 \cos(2\theta)$ and d_{xy} where $\Delta_{xy}(\theta) = \Delta_0 \sin(2\theta)$, with $\theta = \sin^{-1}(k_y/k_F)$, k_y the wavenumber in the y direction and k_F the wavenumber at the Fermi level. For mixed symmetries, we include an isotropic component in the pair potential for $d_{x^2-y^2}$ and d_{xy} wave, so that the pair potential is written as $\Delta_{d+is}(\theta) = \Delta_d + i\Delta_s$, where we have used $\Delta_s = 0.05\Delta_0$ [7]. To describe the transport properties of ferropnictides we consider two models s_{++} and s_{+-} [11], in the first we consider that the phase difference between the two gap is 0 and for the other is π , $\Delta_1/\Delta_2 = \pm|\Delta_1|/|\Delta_2|$, where $\Delta_{1(2)}$ is the pair potential in the band 1(2).

Using the Keldysh formalism, the electrical current can be written in terms of the non-equilibrium Green functions of the system $\hat{G}_{ij,nn}^{\gamma\delta}$, as [14]

$$I = \frac{e}{h} t \int_{-\infty}^{\infty} dE \text{Tr} \left(\hat{g}_{LL}^{+-} \hat{\sigma}_z \hat{G}_{RR}^{--} - \hat{g}_{LL}^{-+} \hat{\sigma}_z \hat{G}_{RR}^{+-} \right), \quad (1)$$

where t is a parameter related with the transmission coefficient for a lead connected to a semi-infinite normal metal ($\Delta = 0$) as $T_N = \frac{4t^2}{(1+t^2)^2}$, the superscripts $+-$ and $-+$ are the branches in the Keldysh space, $\hat{g}_{LL(RR)}$ is the Green function of the lead (superconductor) without perturbation ($t = 0$) in Nambu space

and σ_z is the Pauli matrix. The functions $G^{+-(-+)}$ are calculated from the Green functions of the system perturbed \hat{G}^r solving the Dyson equation [14]. The symbol $\hat{\cdot}$ denotes 2×2 matrices in Nambu space.

In order to calculate the shot noise, we use the spectral density of the electrical current fluctuations, and taking into account the current expression, we obtain a similar equation for the shot noise (2). At zero temperature, non zero voltage and zero frequency, ($T = 0$, $V \neq 0$ and $\omega = 0$) the shot noise is

$$S(0) = \frac{4e^2 t^2}{h} \int dE \text{Tr} \left[\hat{G}_{LL}^{+-} \hat{G}_{RR}^{--} + \hat{G}_{LR}^{+-} \hat{G}_{LR}^{--} \right], \quad (2)$$

Finally, the Fano factor is defined as the ratio between the shot noise at zero frequency and the electrical current $F = \frac{S}{2eI}$. In the tunneling limit the Fano Factor (F) gives information about the effective charge.

3. Results

To calculate \hat{G}_{ij} and \hat{g}_{ij} ($i, j = R(L)$) we use the superconductor surface Green function for HTc superconductors $\hat{g}_S^r(E, k_y)$ [15] with E the energy of the quasiparticle. From $\hat{g}_S^r(E, k_y)$ we obtain the Green function for a one dimensional electrode coupled to a superconductor by

$$\hat{g}_{RR}^r(E) = \sum_{k_y} |p(k_y)|^2 \hat{g}_S^r(E, k_y).$$

this Green function (g_{RR}^r) is affected by the anisotropy of the superconductor pair potential and depends on the thickness for the contact. As we consider a contact that has only one transmission mode, the thickness of the lead satisfies the ratio $\frac{\pi}{k_F} < L < \frac{2\pi}{k_F}$ and then $p(k_y)$ is proportional to the wave number on x direction (k_{0xF}) and to the overlapping between the wave functions on y direction, $p(k_y) = k_{0xF} |\langle k_1 | k_y \rangle|^2$ [16].

In this work we have fixed $\Delta_0 = 20\text{meV}$ and the ratio $\Delta_0/E_F \sim 10^{-1}$, which are typical values for a HTcS.

For voltages $eV < \Delta_0$, general expressions that describes the contribution to the electrical current and the shot noise due to Andreev reflections (AR) are

$$I_A = \frac{16e}{h} t^4 \int_0^{eV} dE \frac{|g_{RR,12}(E)|^2}{|D_2(E)|^2}, \quad (3)$$

$$S_A = \frac{64\pi^2 e^2 t^4}{h} \int_0^{eV} dE \frac{|Im[g_{RR,12}(E)(1 + t^4 D_1(E))]|^2}{|D_2(E)|^2}.$$

Therefore, in this case the Fano factor is given by

$$F = 2e \frac{\int_0^{eV} dE |Im[g_{RR,12}(E)(1 + t^4 D_1(E))]|^2}{\int_0^{eV} dE |g_{RR,12}(E)|^2}. \quad (4)$$

where $D_1 = \det(\hat{g}_{RR,11}^r)$ and $D_2 = 1 + it^2(\hat{g}_{RR,11}^r + \hat{g}_{RR,22}^r - t^4 D_1)$.

3.1. *s* wave superconductors

For this symmetry, we consider $\Delta = \Delta_0$, which means that the magnitude of the pair potential does not depend on k_y . In this case, in the tunneling limit ($t \rightarrow 0$), the shot noise and the electrical current are due only to AR and F is equal to 2 for voltages less than the energy gap. For voltages higher than Δ_0 the quasiparticles transmission increases and the F value decays to 1 (Fig. 2). This behavior is because the AR are equal to 1 for $eV < \Delta_0$ and tend to zero for $eV > \Delta_0$, this result is similar to the obtained for plain junctions [4].

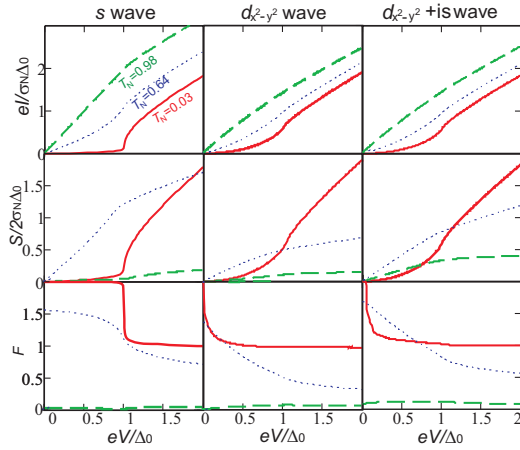


Fig. 2. The electrical current, the shot noise and the Fano factor (First, second and third row) for *s*, $d_{x^2-y^2}$ and $d_{x^2-y^2} + is$ symmetries (First, second and third column) as a function of the voltage for different values of the transmission T_N .

3.2. *d* wave superconductors

For $d_{x^2-y^2}$, the F behavior shows that the effective charge tends to $2e$ only when eV tends to zero, that is because at zero voltage $eV < \Delta(\theta)$ for every θ and the Andreev reflection is the mechanism that transport charge to the superconductor. For $eV > 0$, the quasiparticle probability transmission is not zero for $eV < \Delta_0$ due to the potential anisotropy and it produces a reduction on the effective charge and the Fano factor decreases. For $eV \gg \Delta_0$, the electric effective charge tends to e . For d_{xy} symmetry the Andreev reflection is suppressed and it does not exist effective charge related with AR. Therefore the electrical current and the shot noise are produced only by quasiparticle transmission, for this reason the F is equal to 1 for every

voltage (Fig. 3). This result contrasts with the one obtained for a plain junction [4], where the Fano factor increases from 0 to 1 when the voltage is increased.

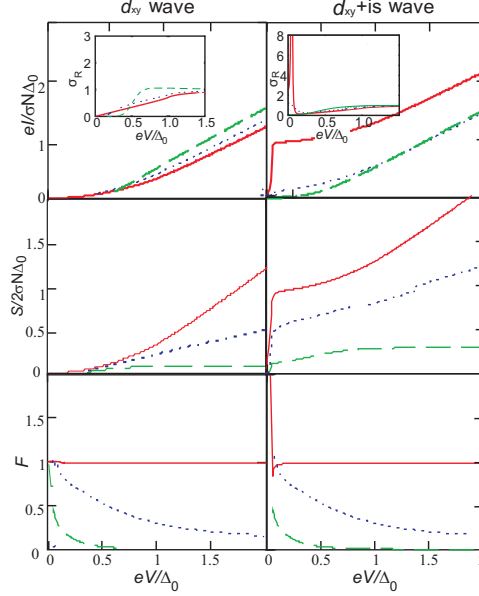


Fig. 3. The electrical current, the shot noise and Fano factor for d_{xy} and $d_{xy} + is$ symmetries as function of the voltage for different transmission values T_N . The insets show the differential conductance where we can see a peak at $eV = \Delta_s$ for $d_{xy} + is$ symmetry.

For mixed symmetries we have found that the effect of Δ_s in F . For $d_{x^2-y^2} + is$ symmetry, $F = 2$ for $eV < \Delta_s$, for higher voltages F decays rapidly and the behavior is the same as the non-isotropic pair potential. For this symmetry, the electrical current and the shot noise are similar to the properties for $d_{x^2-y^2}$ symmetry, the difference could only be seen at low voltages (Fig. 2). For $d_{xy} + is$ symmetry we have found that it appears a new contribution to the current and shot noise due to AR, this can be seen on F because at low voltages the effective electric charge is $2e$ and for higher voltages the d_{xy} symmetry behavior is recovered.

3.3. Multiband superconductors, s_{++} and s_{+-} symmetries

To calculate the Green functions for a lead connected to a multiband superconductor, we mix the Green functions for two *s* wave superconductors by mean of a mixing coefficient α defined as the ratio of probability amplitudes for an electron crossing the interface from the left to tunnel into the first or second band on the right,

$$\hat{g}_{RR}^r(E) = \sum_{k_y} |t(k_y)|^2 [\hat{g}_{\Delta_1}^r(E, k_y) + \alpha \hat{g}_{\Delta_2}^r(E, k_y)]. \quad (5)$$

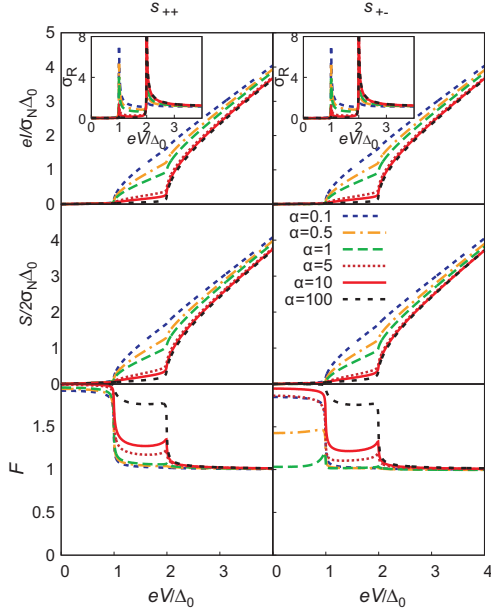


Fig. 4. The electrical current, the shot noise and the Fano factor for s_{+-} and s_{++} symmetries as function of the voltage at tunneling limit for different α values. The insets show the differential conductance.

In figure 4, the electrical current, the shot noise and the Fano factor are shown for different α values in tunnel limit, we have taken $|\Delta_1| = 2|\Delta_2| = \Delta_0$. For s_{++} symmetry when $\alpha = 0.1$ the pair potential dominant is Δ_1 , where the Fano factor is approximately 2 for $eV \leq \Delta_0$ and decays to 1 for $eV > \Delta_0$, when α increases, there is Andreev reflection due to Δ_2 and the Fano factor for $\Delta_0 < eV \leq 2\Delta_0$ takes values between 1 and 2. In s_{+-} symmetry, the pair potential Δ_2 sign produces destructive interference, decreasing the Andreev reflections. The main effect is observed when $\alpha \simeq 1$ where the effective electric charge tends to e , it means that the diffraction on the contact for this α value suppresses the Andreev reflections and the electrical current is due to quasiparticles transmission for $\Delta_0 < eV \leq 2\Delta_0$.

4. Conclusions

We have determined the electrical current, shot noise and Fano factor for a superconductor quantum point contact system using the Green functions and the Keldysh formalism. We have considered five types of symmetries for the pair potential and we found that for d_{xy} symmetry, the contribution to the current and shot noise due to Andreev reflections is null and the effective charge for this symmetry is e at every voltage. In $d_{xy} + is$ symmetry, the AR are recovered and we found that at voltages lower than Δ_s the con-

tribution to the current increases and the effective electric charge is $2e$. At higher voltages, the current and shot noise recover the behavior of the unmixed symmetries. In $d_{x^2-y^2} + is$ symmetry, the isotropic component produces a constant value of $AR = 1$ for voltages lower than the isotropic gap, which means that the Fano factor is 2 for these voltages. For higher voltages the effective electric charge tends to e . On the other hand, for s_{++} and s_{+-} symmetries the Fano factor is affected by the value of the pair potential Δ_1 and Δ_2 , the relative phase between them and the coefficient α that mixes. These results suggest that the Fano factor could be used to find the symmetry in high T_c Superconductors.

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References

- [1] Ya.M. Blanter, M. Buttiker. Phys. Rev. B, 56 (1997) 2127.
- [2] C. Beenakker, C. Schnenberger. Phys. Today, 37 (2003) 56.
- [3] C. Texier, M. Buttiker. Phys. Rev. B, 62 (2000) 7454.
- [4] Y. Tanaka, T. Asai, N. Yoshida, J. Inoue, Phys. Rev. B, 61 (2000) 1902.
- [5] T. Lofwander, M. Fogelstrom, J.A. Sauls, Phys. Rev. B, 68 (2003) 4504.
- [6] B.R. Choi, A. E. Hansen, T. Kontos, C. Homann, S. Oberholzer, W. Belzig, C. Schonenberger. Phys Rev B, 72 (2005) 024501.
- [7] J.R. Kirtley, C.C. Tsuei, J.Z. Sun, C.C. Chi, L.S. Yu-Jahnes, A. Gupta, M. Rupp, M.B. Ketchen. Nature, 373 (1995) 225.
- [8] Y. Wang, H. MacDonald. Phys. Rev. B, 52 (1995) 876.
- [9] A. Biswas, P. Fournier, M.M. Quazibash, V. N. Smolyaninova, H. Balci, R.L. Greene. Phys. Rev. Lett., 88 (2002) 207004.
- [10] Yoichi Kamihara, Takumi Watanabe, Masahiro Hirano, Hideo Hosono. J. Am. Chem. Soc., 130 (2008) 3296.
- [11] A.A. Golubov, A. Brinkman, O.V. Dolgov, I.I. Mazin, Y. Tanaka. Phys. Rev. Lett., 103 (2009) 077003.
- [12] Yan-Yang Zhang, Chen Fang, Xiaoting Zhou, Kangjun Seo, Wei-Feng Tsai, B. Andrei Bernevig, Jiangping Hu, Phys. Rev. B, 80 (2009) 094528.
- [13] Bardeen J. Phys. Rev. Lett., 6 (1961) 57.
- [14] J.C. Cuevas, A. Martin-Rodero, A.L. Yeyati. Phys. Rev. B, 54 (1996) 7366.
- [15] W.J. Herrera, A.L. Yeyati, A. Martin-Rodero. Phys. Rev. B, 79 (2009) 5020.
- [16] W.J. Herrera, J.V. Niño, J. Giraldo, Phys. Rev. B, 71 (2005) 4515.